Quantitative Reasoning

and the mathematical modeling of criminal activities

What is a quantity?

- Object
- Attribute
- Unit
- Measurement process

The box problem

- Starting with an 8.5” x 11” sheet of paper, a box is formed by cutting equal-sized squares from each corner of the paper and folding the sides up.
- Task one: Describe to me how the length of the side of the cutout and the volume of the box covary

- [117.586 Kim] so, how do I even do this? Well as the length varies, the volume's gonna vary - er - It won't matter will it, if the length of the side varies, then the volume's gonna be the same, cause it's the same amount of paper
- [171.895 Kim] The volume's gonna be the same, because, I think, because like, the sheet of paper, if you're using the whole sheet, doesn't the length won't matter cause the volume's like inside?
- [816.008 Kim] the 11 is the length so to find the, like, length of the box.
- [977.846 Kim] So that's [task 1] wrong, right? As the length increases, the volume, cause the length is increasing here, so then the volume decreases, so this would be the opposite. Oh wait, this is the cutout. Wait, now I'm confused. Cause the cutout increases, the length will decrease. So no, that's [task 1] right then

The Train Problem

- A mile-and-a-half-long train enters a tunnel at a constant speed of 20 miles per hour. The tunnel is five miles long. A clever thief is sitting in the front of the train, and he plans to rob a safe located in the back of the train and escape out the back without being seen. How fast must the thief walk (or run) to reach the safe under cover of darkness?
- Identify some quantities that are useful in solving this problem

Quantities

- number of miles from the front of the train to the thief.
- number of miles from the back of the train to the thief.
- number of miles from the front of the train to the end of the tunnel
- number of hours since the front of the train entered the tunnel
- Number of hours until the rear of the train exits the tunnel
- etc.

Identify relationships between quantities

- When would you add?
- When would you subtract?
- When would you multiply?
- When would you divide?

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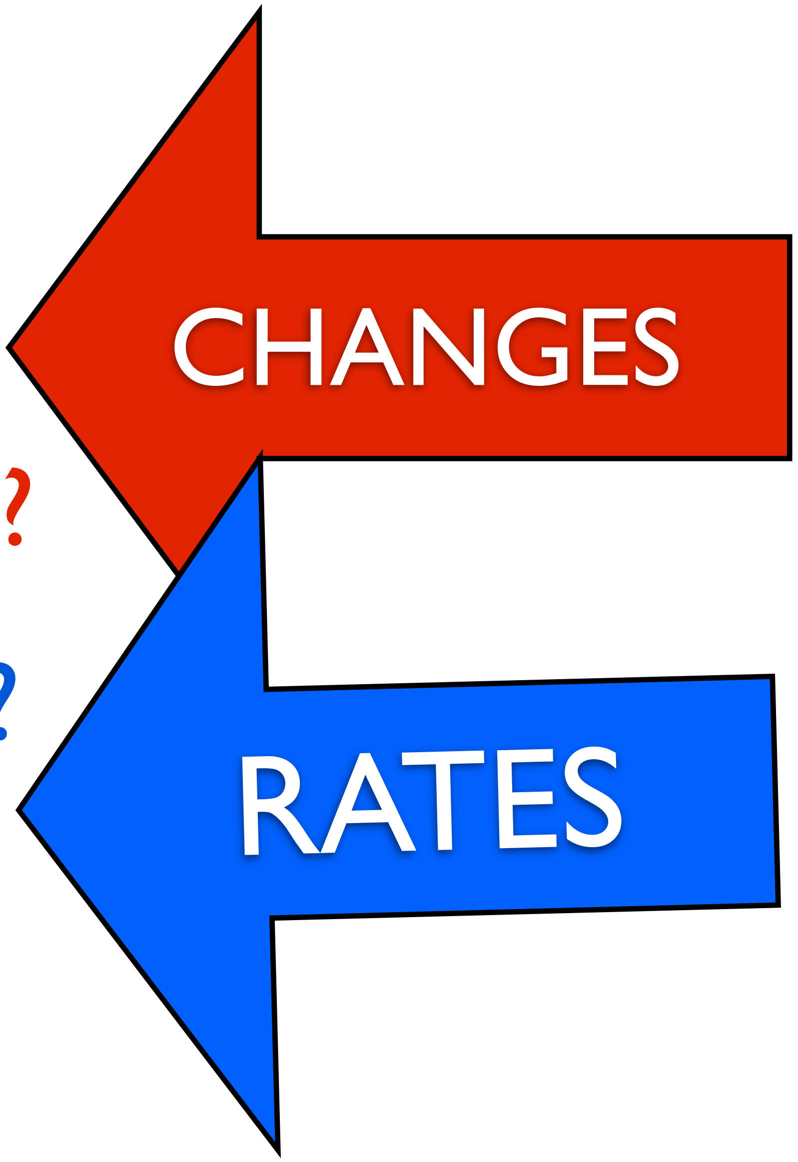
SUMS & DIFFERENCES

RATIOS & PROPORTIONS

Identify relationships between quantities

- When would you add?
- When would you subtract?
- When would you multiply?
- When would you divide?



In inaction

- Robin Banks robs a bank, and drives off. A short time later, he passes a truck stop at which police officer, Willie Katchup is dining. Willie receives a call from his dispatcher, and takes off in pursuit of Robin five minutes after Robin went by. Robin is driving at 60 miles per hour. Willie pursued him at 80 miles per hour. (Paul Foerster 1994)
- Note: 5 minutes is 0.083 hours.

Solution

- number of hours since robin passed the truck stop = x
- number of miles between robin and the truck stop = $60x$
- number of hours since willie left the truck stop = $x-0.083$
- number of miles between willie and the truck stop = $80(x-0.083)$
- number of miles between willie and robin $60x-80(x-0.083)$
- willie catches up to robin $60x-80(x-0.083)=0$
- Time that willie catches robin: 0.332 hours since robin passed the truck stop.

The Problem

- $80(x-0.083)$ or $80x-0.083$?

The Problem

- 80(*x*-0.083) or 80x-0.083?

The Solution

- x=number of hours since robin passed the truck stop
- y=number of hours since willie left the truck stop
- y=x-0.083

The Solution

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- y=x-0.083

In the classroom

In the classroom

Your Assignment

- What rate function characterizes linear growth?
  - [r= constant]
- In what situations would this occur?
  - [driving on cruise control
  - hourly salary (pro-rated)
  - stretching a rectangle
  - $C=2\pi r$]

Your Assignment

- What rate function characterizes quadratic growth?
  - [r=mx+b]
- In what situations would this occur?
  - [constant acceleration
  - falling]

Your Assignment

- What rate function characterizes exponential growth?
  - [r=my]
- In what situations would this occur?
  - [approximate interest
  - approximate population growth
  - making more of itself
  - half-life, population decay]

Reading Assignment

- "Function families and modeling real world phenomena" *Essential Understanding of Functions* pp. 34 -59
    - pages on vertex form (pp. 48-52) are interesting but not relevant, and therefore optional.
    - pages on trig (pp 59-69) are also interesting, but also not particularly relevant (and therefore super-mega-optional).
    - I kind of think this section is pretty good, but as usual, read critically. There are a couple spots, where with a sharp eye, the text is only close to correct, not actually correct.

Last Day Choice

- Exponential growth in detail
  - My specialty
  - Pushes quantity and rate understanding to the limit
- Combining functions (composition, adding functions, etc)
  - Other uses for continuous reasoning
  - Quantity and rate not involved.