Mathematical Modeling and the Nature of Problem Solving

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Upshot: Problem solving is an enormous field of study, where so called "problems" can end up having very little in common. One of the least studied categories of problems is open-ended mathematical modeling research. Cifarelli and Sevim's framework – although not developed for this purpose – may be a useful lens for studying the development of mathematical modelers and researchers in applied mathematics.

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1. Victor Cifarelli and Volkan Sevim's target article bears superficial similarity to some of my own work in studying student's mathematical development (Castillo-Garsow 2010, 2012, 2013; Castillo-Garsow, Johnson, & Moore 2013). We both study the mathematical development of individual students engaging in structured series of mathematical tasks; however, applications differ. My work focuses on the development of particular mathematical tools for modeling, while the authors propose a framework (§46, Table 3) for how a student might expand the scope of a problem to see other problems as similar. The authors' framework is one that I have wanted for a very long time, and I am pleased to see it here.

2. On the other hand, I have always found the phrase "problem solving" to be troublesome. What I would like to do in this commentary is discuss my reservations about the phrase "problem solving" and argue that the results of Cifarelli & Sevim have broader applications to critically understudied areas of mathematics education such as open-ended research modeling.

Distinguishing research modeling and problem solving

3. There is no commonly accepted definition of "problem solving." In §15 the authors' take the perspective of Leslie Steffe, saying that "problem solving" involves a situation, a goal, and that there is "no procedure in the concept to reach the goal." Borrowing from John Dewey (1910: 9), I will call this last aspect a "perplexity" and refer to this meaning of problem solving as the *situation-goal-perplexity* meaning of problem solving. This meaning is compatible with the authors' work, but there are other meanings of problem solving also compatible with the authors' work.

4. For example, Frank Lester defined "problem" to mean "a situation in which an individual or a group is called upon to perform a task for which there is no readily accessible algorithm which determines completely the method of solution" Lester (1978: 54). Lester's definition implies that for a student to engage in "problem solving" there must be an external task; the student has to believe that the task is a problem; and

the student believes that there is a solution (a yet unknown, but recognizable stopping point, which Lester also calls the "outcome"). This *task-problem-solution* meaning of "problem solving" is also compatible with the tasks described in the authors' work, in my own work involving structured tasks, and other literature that focuses on problem solving including Marilyn Carlson and Irene Bloom (2005), and George Polya (1973).

5. The *task-problem-solution* meaning differs from the *situation-goal-perplexity* meaning in two ways. The first is the distinction between a "solution" and a "goal." A goal may encompass finding a solution, finding multiple solutions, making progress towards a solution, or even a goal entirely unrelated to solutions (such as improved understanding). Two examples are the "Open-ended" problems such as those described by Jerry Becker and Shigeru Shimada (1997) or the "Model Eliciting Activities" described by Richard Lesh and Guershon Harel (2003). These tasks do not typically have a single recognizable stopping point.

6. The second distinction is between "task" and "situation." Lester (1978: 54) describes a task as being externally imposed, and in all the above cited examples, the task is determined by the instructor or researcher, rather than by the student. However the situation-goal-perplexity meaning encompasses a much larger class of problems. I will illustrate with an example from Mitchel Resnick (1997: 68–74).

7. Resnick describes two high school students, Ari and Fadhil, who were working with the agent based modeling program *StarLogo* at the same time they were enrolled in driver's education. Ari and Fadhil developed a curiosity: they wanted to know what caused traffic jams. Using *StarLogo*, Ari and Fadhil developed several simulations of drivers on a highway, and explored driver behaviors that contributed to or eliminated traffic jams. Although Ari and Fadhil did not succeed in controlling their simulated traffic jam, they discovered quite a bit about traffic jam behavior: that traffic jams moved as waves in the direction opposed to traffic, and that starting all cars at the same initial speed did not prevent a traffic jam, so long as the cars were unevenly spaced.

8. Both the case of Marie and the example of Ari and Fadhil involve modeling in the sense of students thinking about quantities, measurements, and relations between quantities. Both involved imagining fictional situations and describing them mathematically. However, Ari and Fadhil's project differs from Marie's work in that Ari and Fadhil were pursing their own curiosities. Unlike Marie, Ari and Fadhil were engaged in *mathematical research*.

9. This activity is what I mean when I say "research modeling," and based on Ari and Fadhil's example, I would propose three criteria for identifying research modeling activity: (1) The problem originates with the student, (2) based on their non-mathematical experience; and (3) the goal of the activity is understanding, not a solution. This type of experience is compatible with Steffe's situation-goal-perplexity meaning of problem solving, but it differs greatly from the task-problem-solution meaning. In contrast, Marie's work was contained entirely within the task-problem-solution meaning.

Research modeling in MTBI

10. Another example of research modeling comes from my own work with the Mathematical and Theoretical Biology Institute (MTBI) and the Institute for Strengthening the Understanding of Mathematics and Sciences (SUMS; Camacho, Kriss-Zaleta & Wirkus 2013; Castillo-Chavez & Castillo-Garsow 2009; Castillo-Garsow, Castillo-Chavez & Woodley 2013). MTBI is a summer Research Experience for Undergraduates (REU) in mathematical biology, and SUMS is its partner summer program for high school students. By encouraging students to return as advanced students, peer-mentors, tutors, and instructors, MTBI/SUMS serves as a mentorship pipeline in the mathematical sciences that reaches from high school to tenure.

11. MTBI/SUMS has been extraordinarily successful recruiting underrepresented minorities (URMs) and developing them into mathematical researchers (Castillo-Garsow, Castillo-Chavez & Woodley 2013). As of February 2012, 69 U.S. citizen alumni of MTBI have completed a Ph.D. and 54 (79%) were URMs. Most of these degrees have been recent. Based on job posting data, Castillo-Garsow, Castillo-Chavez, & Woodley (2013) estimated that MTBI alumni have been awarded between 1.6% and 4.3% of recent Ph.D. in applied mathematics and 50% of all mathematical biology Ph.Ds awarded to U.S. Latin(o/a)s since 2005.

12. MTBI is an 8 week program. The first three and a half weeks consist of lectures and homework in population biology. The students study difference and differential equations, statistics, stochastic models, agent based modeling, and computer simulation. The structure of these weeks roughly follows Fred Brauer and Carlos Castillo-Chavez (2012), supplemented by guest lecturers. This portion of the program follows the task-problem-solution model.

13. The second half of the program is student-driven group research projects. In MTBI's initial prototype year (1996), the projects were assigned as tasks. In subsequent years, students were expected to design their own projects, while graduate students and faculty served in advising roles. This portion of the class follows the situation-goal-perplexity model. Because students choose their topic of study, they frequently know more about the situation than the mentors. The mentors contribute mathematical and modeling experience, but rarely situational knowledge (Camacho, Kriss-Zaleta & Wirkus 2013; Castillo-Chavez & Castillo-Garsow 2009).

14. One of the most notable aspects of these student projects is their variety. Although trained as population biologists and mathematical epidemiologists, students have applied these techniques to a broad range of interests including: the three strikes law (Seal et al. 2007), gang recruitment (Austin et al. 2011), education (Boyd et al. 2000; Diaz et al. 2003), immigration (Catron et al. 2010), political third party formation (Romero et al. 2011), mental illness (Daugherty et al. 2009; Dillon et al. 2002;), pollution (Burkow et al. 2011), obesity (Evangelista et al. 2004), drug use (Ortiz et al. 2002; Song et al. 2006), and even MTBI itself (Cristoso et al. 2010). SUMS students work with a reduced curriculum in a similar environment, and similarly extend their mathematical biology tools to interests such as traffic, aerodynamics, and education.

15. MTBI highlights a critical distinction between these examples of research modeling (Ari and Fadhil, MTBI/SUMS) and Marie: the scope of the abstraction. Marie abstracted a scheme of a particular task type and was able to assimilate new tasks to that scheme, as well as generate new tasks of that type. These research modeling students abstracted mathematical schemes that assimilate not tasks, but their own interests. To MTBI students, bulimia and tick-host interactions are the same "type" of problem (both applications of systems of ordinary differential equations). The way they see their own world has been mathematized. For any curiosity they have, MTBI students will check if the mathematical tools they learned are appropriate to exploring that curiosity.

Sequencing tasks for building modeling

16. It is in the context of this distinction between engaging in tasks and exploring a curiosity that I want to speak about the potential ramifications of the Cifarelli & Sevim's work. Julie Gainsburg (2006) suggested that these two classes of activities have very different challenges for students, and that work in K-12 tasks may not adequately prepare students for "adult" modeling. A question I want to explore is: Can sequences of tasks develop problem solving skills that encompass not just task-problem-solution problem solving, but also the larger world of situation-goal-perplexity problem solving, including research modeling?

17. Anecdotally, the answer appears to be yes, although that "yes" may be qualified by yet unexplored factors. MTBI/SUMS students begin working with structured tasks and later appear to assimilate a variety of situations to the schemes they have abstracted from these structured tasks. The process could very well be quite similar to the process of recognition, re-presentation, and reflective abstraction that the authors describe in §46.

18. However MTBI/SUMS and similar REUs are little studied by mathematics educators, and the reasons for MTBI's success are not well understood (Castillo-Garsow, Castillo-Chavez & Woodley 2013). To my knowledge, the closest that anyone has come to a radical constructivist study of a mathematical biology REU is Erick Smith, Shawn Haarer and Jere Confrey's (1997) study of a graduate level mathematical biology class. So while it is possible that MTBI students follow a similar trajectory of assimilation as Marie, this development has never been observed, because no radical constructivist has been around to assimilate it. Furthermore, it is unclear exactly how the tasks MTBI/SUMS students engage in are different from the tasks that Marie engaged in. The resulting abstractions appear to be different, but what are the reasons for those differences?

Conclusions

19. Since its foundation constructivism has had a history of careful study of students' understandings by way of precisely designed tasks (e.g., Piaget 1967, 2001). These studies have provided tremendous insight into the development of the mathematics of students. But the field has grown enough that it is time for constructivists to give up some of that control, and apply our lenses to the careful study of the mathematics of

researchers, including student researchers. We need to extend our studies beyond the set of task-problem-solution problems to the broader set of situation-goal-perplexity problems by making a large-scale concerted effort to understand problem solving in the relative complement.

20. The large-scale study of mathematical modeling research programs such as REUs by multiple groups of radical constructivists is critical both to the development of these REUs, and to the advancement of the constructivist study of mathematics education. As it stands, we simply do not know how mathematical researchers are formed, or how the development of mathematical researchers might be promoted in the lower grades. I would urge myself, the authors, and all interested readers to form collaborations with mathematical scientists and applied mathematicians for the purpose of studying precisely how mathematical researchers are formed using tools such as Cifarelli & Sevim's framework.

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