

THE DESIGN OF DIDACTIC OBJECTS FOR USE IN MATHEMATICS TEACHERS' PROFESSIONAL DEVELOPMENT

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In this paper we provide an analysis of the work of a video-based case development team whose goal was to produce didactic objects to be used the professional development of secondary mathematics teachers. In order to generate artifacts for use in the creation of the cases, the research team conducted a classroom intervention in an Algebra I classroom. The daily videotapes, copies of all the student work, and interviews with the teacher comprised the resources for the case development effort. As design researchers, we engaged in interactions of design and research as we tested and refined our development efforts. An important aspect of the work is its focus on the unifying mathematical concept of covariation.

Introduction

In this paper we analyze the work of the Case Design Project [Cadept]¹ that is part of the TPCC [Teachers Promoting Change Collaboratively]² Project. As background, the larger TPCC project entails multiple stages of research and development employing a design research perspective (cf. Brown, 1992; Cobb, Confrey, diSessa, Lehrer & Schauble, 2003). Following this design orientation, the TPCC research team first addressed the need for a strong mathematical basis for teachers by engaging them in a series of three graduate-level courses called Extended Analysis of Functions [EAF]. The mathematical content of the EAF courses focused on developing a coherent understanding of the secondary mathematics curriculum from the complementary perspectives of (1) functions and quantitative relationships (with covariation being a foundational idea for both) and (2) representational equivalence. In addition, these courses were designed and implemented as a model of the type of interactions and discussions that supported the mathematical thinking being developed during the course.

Next, the work of the grant was extended to the school setting as groups of teachers enrolled in the EAF courses met weekly in the format of Professional Learning Communities [PLC's]³ with the goal being that of reflecting on practice as it related to the big mathematical ideas of the courses. Our intent was that PLC meeting agendas would be tightly linked with issues that emerged in the EAF courses. As a result, the relevance of the issues to the teachers' classroom practices provided the link between the courses and teachers' classrooms (cf. Zhao & Cobb, 2006). Each PLC was assigned a facilitator from the TPCC project with the expectation that within a three-year period each PLC would become self-facilitated. The appointed facilitator initially set the meeting agendas and conducted these meetings. The means of support used to initiate teacher reflections typically included (1) teacher developed student interviews, (2) Japanese-style lesson study or (3) sharing a self-recorded video of a teacher in the PLC teaching a particular lesson in his classroom.

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During the preliminary analysis of both the EAF courses and the PLC's, the research team realized that its work was not supporting the teachers' ability to formulate an *image*⁴ of the practices that were being promoted in the project. The teachers also had difficulty imagining the kind of interactions that would support students in understanding the big mathematical ideas from the EAF courses. Examples of teachers' difficulty in understanding the ideas promoted in the project emerged particularly during PLC sessions. When discussing student interviews the teachers conducted, their focus was on students' answers and not how the students thought about obtaining their answers. When a teacher would share a video recording of a teaching segment from her classroom, other teachers' were (1) either hesitant to share their opinion so as not to offend the teacher or (2) focused on classroom management issues. In their discussions, little focus was placed on student thinking.

In order for the TPCC research team to better understand these perceived difficulties and then create conversations that would address the difficulties, it decided to generate artifacts for use in the professional development settings. The goal was to create video-based cases in which teaching took the form of a long-term coherent approach to significant mathematical ideas in a classroom setting where students' current ways of reasoning were at the forefront of decision making and planning. As a result, the Case Development Project [Cadept] was developed. The goal of Cadept was twofold. First, the members of the Cadept design team wanted to create potential didactic objects⁵ (cf. Thompson, 2002) that could be used with teachers to reflect on teaching in relation to student learning; and second, these objects needed to provide comprehensive understandings of the struggles teachers encounter as they attempt to implement what they understand to be the big mathematical ideas in their classrooms.

In order to generate the artifacts necessary for creating the potential didactical objects, the TPCC research team determined that it needed to conduct a classroom intervention⁶ with one teacher in order to produce a record of her attempts to teach a conceptually oriented course. The team selected a ninth-grade Algebra I course for non-honors students —students with whom the team would later work in Geometry and Algebra II. The teacher, whom we call Augusta⁸, was a full participant in the classroom intervention. Augusta was chosen as the teacher for the experimental classroom because she was comfortable taking risks and trying out instruction for which the eventual outcome was unclear. She also was willing to collaborate with the TPCC research team in the process of designing the course. In addition, Augusta's principal was eager to have this project in his school. This, therefore, removed some potential institutional constraints. During the year of the intervention, each class session was videotaped for two purposes: for our own understandings of the struggles that teachers face, and for potential use in generating artifacts.

The development of the cases was an iterative process of ongoing analysis, modification and refinement. Much like Simon's (1995) *Mathematics Teaching Cycle* the TPCC research team engaged in both *meta* and *mezzo* levels of design and revision during which it focused on both the design of the professional development courses for the teachers and the design of activities for the classroom.

Against this background, we next document the evolution of the need for the classroom design intervention. We follow by documenting the research and design cycle that was employed in the development of video-based cases from Augusta's classroom. We then give a summary of the current state of our work. We conclude with an analysis that provides implications of our work for other university collaborators and the field at large.

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The Evolution of the Need to Design a Classroom Intervention

As noted earlier, the need to generate classroom artifacts emerged from ongoing analyses of work in both the EAF courses and the PLC's. Further, the teachers' curricular knowledge—their understandings that corresponded to textbook material they felt compelled to teach and their image of problems that students must know how to solve—overwhelmed their ability to imagine teaching a series of lessons that developed ideas relationally, coherently, and longitudinally independent of their text. As a result, the research team decided to conduct a classroom design intervention with one teacher to produce a record of that teacher's attempts to teach a conceptually oriented course.

It was conjectured that the record of Augusta's classroom would provide a data source for us to use in documenting the process of both the teacher and her students' learning conceptually oriented mathematics. In addition, the team conjectured that the struggles emerging as part of this learning would also be documented. As a result, our design was focused on (1) Augusta's reconceptualization of Algebra I, (2) students' mathematical learning, (3) appropriate instruction to teach what Augusta reconceived so that students could learn it, and (4) the means of support for Augusta's transformation.

The Artifact Collection Process

In addition to the daily-videotaped classes, the TPCC research team also created an electronic record of the lesson designs for the year, videotaped daily debriefing sessions with Augusta after the class period, audio recorded weekly collaborations with Augusta, made copies of all student work, and videotaped student interviews with the research team. This extensive data corpus not only provided the resources for use in understanding the difficulties associated with teaching conceptually oriented mathematics, but it also provided artifacts that could be used in the design of potential didactic objects. As a result, the Cadept team's initial design conjectures for the artifacts was focused on (1) instances of Augusta's coming to conceptualize an instructional sequence to promote students' mathematical learning, and (2) means of support for Augusta's transformation. It is therefore important to note that the intention of the design was not to focus on Augusta per se; but rather on the generation of artifacts which could be used to focus other teachers' attention on Augusta's reconceptualization of her teaching practices. The motivation for focusing the design of the case study on Augusta's reconceptualizations and teaching practices were based on the observations made from the PLC and EAF courses.

This was a highly interventionist and time-intensive process. As part of this process, frequent exchanges occurred between Patrick Thompson and Augusta both after class and during their Saturday planning sessions. The goal of these exchanges was to support Augusta's ability to reason logically with the innovative materials while using the student's ways of reasoning as an important aspect of planning. In addition, these meetings assessed the effectiveness of the materials in developing student thinking and Augusta's understanding of these materials. The meta-level goal of these exchanges was to gain insight into Augusta's difficulties as she was teaching with these innovative materials while getting her input into subsequent design.

The Design of Video-based Cases

As the TPCC research team reflected on the unfolding "story" from Augusta's classroom and on the changes in Augusta during the teaching of the Algebra I course, it saw the video as a potential source of didactic objects for professional development. The team determined that the Swars, S. L., Stinson, D. W., & Lemons-Smith, S. (Eds.). (2009). *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Atlanta, GA: Georgia State University.

video would make a compelling case for other teachers. In particular the research team identified appropriate instances from the classroom video to choose as segments that would form the basis of the cases. The Case Design Project [Cadept] was therefore developed to create a series of video-based artifacts to be used in a professional development setting. These video artifacts would be part of a larger package of artifacts that included problem sets for the teachers, curriculum critique and development, and analysis of student work. These materials were being designed to provide opportunities for the teachers to reflect on their practices by examining Augusta's classroom.

The initial exploration of the data yielded six potential cases: (1) covariational reasoning, (2) linear functions, (3) systems of equations, (4) sums of functions, (5) factoring and polynomials, and (6) quadratics. As the design team worked, each case required condensing the classroom video into sequences of short video stories that could be supported with additional resources from Augusta's classroom. The video was edited both to make these stories of practical viewing length, and also to emphasize specific plots. These plots involved the students struggling with mathematical ideas, the teacher struggling with implementing those ideas, the development of discourse in the classroom, and the cognitive development of the students including significant mathematical benchmarks and shifts.

Pilot Studies as Part of the Design Cycle

As noted, the design team took a *design research* perspective in its development process. As a result, selected video segments were piloted with teachers throughout the development process. For example, the third EFA course served as one pilot study. The goals of this study were to draw teachers' attention to student thinking and the subtleties of covariational reasoning and instruction. Initially, the teachers in the functions course did not focus on content nor student understanding. Their original focus was on Augusta's classroom management. Their assessment of the success of the lesson was directly related to how well the students' behaved. Moreover, the teacher's focus was on Augusta rather than on the students she was teaching to. They did not discuss the students' thinking, nor notice the role of Coordinating Quantities Tool⁸ in the lesson. They viewed the tool as a "nice activity." When they broke into groups to watch individual video clips, their discussions indicated that they did not have a theory of learning or a notion of an epistemic student. It was only after discussion and probing by Patrick Thompson (the teacher of the functions course) that the teachers attempted to focus on student thinking. As a result, were able to articulate evidence as they built models of student thinking. As an example, they were able to examine one student's use of the Coordinating Quantities Tool to make conjectures about her ways of reasoning about the coordination of the two quantities. Also, the teachers' image of covariation changed. They shifted from shape thinking⁹ to covariational reasoning.

Throughout the study, it was apparent that simply changing the curriculum or improving teacher's content knowledge would not provide a sufficient stimulus for change. These issues must be addressed in the context of exploring classrooms (cf. Zhao, 2007). Zhao makes a strong argument for the necessity of "conceptualizing the relations between teachers' learning in the setting of professional development and their instructional practices in the classroom" (p. 3). She argues that

[r]egardless of researchers' continuous efforts to design and support teachers' professional development, changes in classroom mathematics instruction do not always occur as intended. Thus, an immediate and pragmatic challenge posed to

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teacher educators necessarily involves how to design professional development activities so that teachers can relate what they learn to their classroom practices and, as a result, become willing to engage in changing their current ways of teaching. (p. 4)

In order to address this conundrum, the design team therefore focused its pilot efforts on understanding the relation between the classroom-based video artifacts and the teachers' reactions with respect to their practice.

As a result, the research team constructed an epistemic model of teachers' images of the classroom. The model included the fact that teachers would not attend to student thinking without readily available evidence and someone pressing them to hold to that evidence. Further, the text emerged as the dominant resource for planning. Also, possible distracters in video emerged. For example, teachers paid more attention to classroom management issues than to the intended focus of the video segments. The epistemic model of teachers' image of Augusta's classroom was used as a factor in choosing the story that was to be told of the important issues to be discussed around the video based cases. As a result, selected video segments were continually piloted throughout the development process. The iterative process was crucial in the success of our final design.

Results of Analysis, Conclusions and Implications

Numerous scholars in the field of mathematics education have advocated the importance of teachers having strong knowledge of the content they teach (cf. Ball, 1990; Bransford, Brown, & Cocking, 2000; Grossman, 1990; Ma, 1999; National Research Council, 2001; Schifter, 1995; Sowder, et al., 1998). This sentiment is echoed in the *No Child Left Behind* legislation that articulates a demand for highly qualified teachers who display mastery of subject matter. There is, in fact, general agreement in both the political and educational arenas that knowledge of content is a necessary condition for an effective mathematics teacher.

However, we have learned that this knowledge is necessary, but not sufficient. Being able to take newly acquired knowledge and transpose it into a new image of teaching is challenging at best. As we have noted, teachers must also develop *images of good teaching*. These images must be grounded in the teaching of significant mathematics where student thinking guides instructional decision-making. Here we have argued that the investigation of a well-designed video-based case can provide the context in which to make explicit the complexities involved in innovative mathematics classrooms. In doing so, we provide a context in which to examine the use of video-based cases in supporting teachers' professional growth, including their understandings of issues of both mathematical content and pedagogy. It is in this context that opportunities for teachers to reflect on their teaching practices arise.

However, investigations of classrooms provide both potential resources and pitfalls. Teachers view classrooms through the lens of their prior beliefs, thereby negating any efforts for issues of teaching and learning to be made explicit through their observation. For this reason, teachers' discussions of classrooms often take on the characteristics of "storytelling" during which the teachers in professional learning communities share their interpretations of accounts from the classrooms. The judgments they make about what they observe and experience can become traps that prevent professional growth. Overcoming this can be a formidable task. We therefore cannot assume that the issues that are focused upon during collaborations will be made explicit and then acquired naturally through teaching.

However, the effectiveness of video-based, multi-media cases has been documented by Swars, S. L., Stinson, D. W., & Lemons-Smith, S. (Eds.). (2009). *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Atlanta, GA: Georgia State University.

Richardson and Kyle (1999) who state that “the use of multimedia cases significantly impacts teachers’ cognitions” (p. 131) They note that the power lies in the cases’ ability to present “a visual, moving picture of teaching in a real-life classroom” (p. 136). Video-based cases also allow easy access to numerous facets of the classroom to facilitate in-depth investigation of issues of content, the teacher’s decision-making process and students’ diverse ways of reasoning. Through their investigation and critique of a case, teachers have the opportunity to develop and refine their skills in critiquing, evaluating and creating learning experiences. Their image of teaching is changed as a result.

The critical aspect of this process is the guiding and framing of the experience by the facilitator. Just as we view the role of the teacher as critical in supporting students’ developing understandings of mathematics (or any other content area), we view the role of the facilitator as critical in supporting teachers’ understandings of what it means to teach mathematics effectively. We do not believe that the cases are transparent carriers of meaning. Nor do they have agency. They are, in fact, tools to be used in the course of teacher collaborations (cf. Kaput 1994; Miera, 1998; van Oers, 2000). The goal is then to create the settings in which these cases can become genuine didactic objects. For this reason, our next cycle of design and research will focus on the development of facilitators’ guides. However, like Carpenter and colleagues (Carpenter, Blanton, et al), we do not believe that forms of professional development can be codified and handed over as a means of scaling up. Therefore, the next steps in our design process will involve cycles of design and revision while working closely with other university collaborators.

Although our process is still ongoing, we claim to have documented evidence to support the following guiding principles:

1. *The thoughtful design of a video-based case is essential in creating effective means of supporting teacher professional growth and development because it provides a bridge between the professional development setting and the classroom.*
2. *Video-based cases must support the larger goals of any collaboration.*
3. *The strength of a video-based case is limited by the quality of instruction and the nature of the student discourse captured in the video.*
4. *Video-based cases can only become didactic objects when thoughtful consideration has been given to their design and use.*

In order for the cases to meet these guiding principles and therefore support teachers’ ability to re-conceptualize their practice, they need to provide resources to support the teachers’ construction of an image of a conceptually oriented mathematical conversation with students. A conceptual conversation is one that has a diminished emphasis on technique and procedure while having an increased emphasis on images, ideas, reasons, goals, and relationships. People conversing conceptually speak in ways that make their meanings, ideas and ways of thinking clear to others in the conversation. To avoid speaking in ways that could possibly hide their meaning, these individuals are aware of possible interpretations of their words another may have which are different from the meaning that they intended. The design, testing and refinement of our cases and the supporting material can therefore provide this opportunity. This is significant in that it offers a means of supporting teachers’ transitions in professional development setting.

Endnotes

1. The Case Design Project Team [Cadept] is composed of Kay McClain, Scott Adamson, Ted Coe, Carlos Castillo-Garsow, Sharon Lima and Patrick Thompson.

2. The research team is composed of Patrick Thompson (Principal Investigator), Scott Adamson, Ted Coe, Carlos Castillo-Garsow, Sharon Lima, and Kay McClain. Research reported in this paper was supported by National Science Foundation Grant No. EHR-0353470 under the direction of Patrick W. Thompson. Any conclusions or recommendations stated here are those of the authors and do not necessarily reflect official positions of NSF.

3. We use the term Professional Learning Communities to denote the cohorts of teachers within the schools who met on a weekly basis to discuss issues related to the college course. An analysis of the development of the cohorts into communities is beyond the scope of this paper. We therefore realize that we are taking liberties with the term *community* and do not intend to imply that we have conducted analyses to confirm that these cohorts actually transformed into communities (cf. Dean, 2005; Wenger, 1998).

4. By “image” we build from what Maturana (1978) describes as a conceptual system through which we may anticipate another system’s behavior. These images are highly related to what Cobb has in mind when he speaks of an envisioned practice as a goal of instructional design.

5. Elsewhere, we have used the phrase didactic object to refer to “a thing to talk about” that is designed with the intention of supporting reflective mathematical discourse (see Thompson, 2002). In doing so we note that objects cannot be didactic in and of themselves. Rather, they are didactic because of the conversations that are enabled by someone having conceptualized them as such. In this sense, a didactic object is a tool, but one designed to produce desirable conversations.

6. We make a distinction between a classroom intervention and a classroom teaching experiment or a classroom design experiment. In the intervention, the goal of the TPCC research team was to elicit certain ways of reasoning and certain struggles from both the teacher and the students.

8. The Coordinating Quantities Tool (or finger tool) makes use of the index finger on each hand by asking students to track the changes in the quantity of the independent variable with a horizontal movement while simultaneously tracking the quantity of the dependent variable in a vertical movement.

9. Thompson makes a distinction between “shaping thinking” and covariational reasoning. In shape thinking, students can imagine the shape of a graph from the scenario such as the distance of a bungee jumper from a bridge as he bounces back and forth. The graph is then a static trace of an event that has occurred. Covariational reasoning requires the student to think about how two quantities vary in relationship to each other or co-vary.

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