Part I – Geometric Growth

1) Pennies on a Chessboard

Imagine a chessboard of 64 squares. Place a penny on the first square. On the second square place twice the number of pennies. On each square, double the number of pennies on the previous square.

a. How many pennies will there be on the 7^{th} square?

b. How many pennies will there be on square *n*?

c. How many pennies will there be on square 4.3?

d. Sketch a graph of the number of pennies on each square, keeping in mind your answers to a-c.

e. Write an equation of the number of pennies *y* on square *x*. What is the domain of x?

2) Elephants

Every spring, on April 7^{th}, new elephants are born. Each elephant gives birth to two new elephants every year. Scientists began an elephant census on April 8^{th}, 1973, which they call year 0.

a. A herd has 7 elephants at year 0. How many elephants will there be at year 1?

b. Year 2?

c. Year 3.4?

d. Sketch a graph of the number of elephants at year *x*, keeping in mind your answers to a-c.

e. Let y be the number of elephants at year x. Explain why the equation $y = 7(3^x)$ will not accurately predict the number of elephants.

f. The notation $\lfloor k \rfloor$ means the largest integer less than or equal to k. Using this notation, correct the equation from part e.

3) Bacteria

The mass of a colony of bacteria in a petri dish triples every day. The dish begins with 0.2 grams of bacteria on day 0. However, the bacteria do not all divide at the same time, the growth happens throughout the day. Let y be the mass of the colony on day x.

a. How much will the colony mass at day 1?

b. How much will the colony mass at day 6?

c. The growth factor from day x to day $x+k$ is defined as $\frac{y_{x+k}}{y_x}$.

   i. What is the growth factor from day 0 to day 1?

   ii. What is the growth factor from day 1 to day 6?

   iii. Propose a growth factor for day 1 to day 1.5. Justify you choice.

d. How much will the colony mass at day 1.5?

e. How is the bacteria situation different from the elephants?

f. Sketch a graph of the mass of the colony at year x, keeping in mind your answers to a-d.

g. Write an equation for the mass of the colony y at year x

Part 2 – Compounded growth

1) Linear interest

A student run bank offers to pay interest at a rate of $8 per year on its accounts.

a. Carlos invests $100 at the student bank. Write an equation predicting Carlos' bank account balance y at x years.

b. Gabriela invests $500 at the student bank. Write an equation predicting Gabriela's bank account balance y at x years.

c. Graph Carlos and Gabriela's bank accounts over time on the same set of axes.

d. Nohora invests $n at the student bank. Write an equation predicting Nohora's bank account balance y at x years.

e. How could Gabriela invest the same amount of money in the student bank, but earn more?

2) Simple Interest

Taking into account your suggestions, the student run bank decides to offer a new growth rate based on the initial investment. The new student bank policy pays interest at a rate of 8% of your initial investment per year.

a. Carlos invests $100 at the student bank. How fast is his bank account growing?

b. Gabriella invests $500 at the student bank. How fast is her bank account growing?

c. Graph Carlos and Gabriela's bank accounts over time on the same set of axes.

d. Nohora invests \$n at the student bank. Write an equation predicting Nohora's bank account balance *y* at *x* years.

e. Gabriela has a plan to start with the same \$500 but earn money faster. She plans to take all her money out of the account every 1/4th of a year, and open a new account with the money. Why will Gabriella's plan earn her money faster?

3) Compound Interest

Taking into account your suggestions, the student run bank decides to offer a new policy of updating your growth rate every 1/4th of a year. The new student bank policy pays interest at a rate of 8% of your quarterly balance per year.

a. Gabriela invests \$500 in her account. How much money will Gabriela have at 0.1 years? How fast is Gabriela's account growing?

b. How much money will Gabriela have at 0.25 years?

c. How much money will Gabriela have at 0.3 years? How fast is Gabriela's account growing?

d. How much money will Gabriela have at 0.6 years? How fast is Gabriela's account growing?

e. Sketch a graph of Gabriela's account balance over the first year.

f. Write an equation predicting Gabriela's account balance on a domain of $0<x<1/4$ years.

g. Write an equation predicting Gabriela's account balance on a domain of $1/4<x<2/4$ years.

h. Write an equation predicting Gabriela's account balance on a domain of whole quarters of a year only. (Hint: distributive property).

i. Write an equation predicting Gabriela's account balance on a domain of $k/4<x<(k+1)/4$ years.

j. How could Gabriela modify her strategy to earn more money with the same investment?

k. How would you modify the equation in part *h* for updating the rate *n* times per year?

l. Optional Fun Time: Use $\lfloor k \rfloor$ notation to write an equation for Gabriela's account balance all in one piece on a domain of $0<x$.

4) Continuous Interest

Taking into account your suggestions, the student run bank decides to offer a new policy of updating your growth rate continuously. The new student bank policy pays interest at a rate of 8% of your current balance per year.

a. Gabriela invests $500 in her account. Sketch a graph of Gabriela's account balance over time.

b. Why can't Gabriela modify her investment strategy to earn more money?

c. Use your equation from part 3k and the formula $\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n = e^r$ to find an equation for Gabriela's account balance over time.

d. Why is it OK to use the equation from 3k instead of the more accurate equation from 3i?

Part III – Per-Capita rate of change

1) A marine biologist calculates that a kilogram of plankton produces more plankton at a rate of 0.05 kilograms per month.

a. How fast would 2 kilograms of plankton produce plankton?

b. How fast would 3.6 kilograms of plankton produce plankton

c. How fast would y kilograms of plankton produce plankton?

d. Write an equation that relates the rate of plankton r growth to the number of kilograms of plankton y.

e. Graph the relationship between the rate of plankton growth and the number of kilograms of plankton.

2) Examine your graph from problem 1.

a. As the number of months increases, what do you expect to happen to the number of kilograms of plankton?

b. Place the finger of your right hand on the horizontal (y) axis, to show some number of kilograms of plankton at 0 months. Imagine pressing start on stop-watch and move your finger to show the effect of time on the mass of plankton. Describe how your finger moves.

c. Place a second finger of your left hand on the vertical (r) axis, to show a rate of kilograms of plankton per month at y kilograms. Imagine pressing start on a stop-watch and move both fingers to show the effect of time on the mass of plankton. Describe how your fingers move.

d. What is the effect of the mass finger on the rate finger?

e. What is the effect of the rate finger on the mass finger?

f. Sketch a graph of the mass of plankton over time.

Part IV - Waiting time

1) Robot Army

   Imagine a robot that builds copies of itself. The robot takes 5040 hours to make a copy. Once the copy is made, both robots immediately begin working together to make a third copy twice as fast. When the third copy is made, all the robots begin making a fourth copy, and so on.

   a. How long does it take for there to be three robots?
   b. How long does it take for there to be four robots?
   c. At 10,000 hours, how many robots are there?
   d. Sketch a graph of the number of robots over time.
   e. Although the growth of the robots becomes approximately exponential, There is no exponential curve that will fit through the beginning of each step. Why do you think that is?

Part V – Doubling Time

1) Imagine that an initial investment of $n grows continuously according to the function $y = n(1.071773)^x$, where y is the value of the account at x years after the initial investment was made.

a. What value will y have when the investment has doubled?

b. What value will x have when the population has doubled?

c. I've rewritten the function above as $y = n(2)^t$. In order for the investment to have the same value at every point in time, what must the units of t be?

d. The current U.S. National debt is 13 trillion dollars. Starting from a 1 dollar investment, how many doublings would it take for the investment to reach 13 trillion dollars?

e. How many years would it take for a 1 dollar investment to reach 13 trillion dollars?

f. How many years does it take the investment to earn the first 6.5 trillion dollars?

g. How many years does it take the investment to earn the last 6.5 trillion dollars?

h. Without using a calculator, graph the function $y = 10(1.071773)^x$ over 50 years. Identify the coordinates of three points on the graph and explain how you know their values.

Part VI – Stochastic Process

1) Begin with one six-sided die in your "die pool" and play the following game:

Roll all the dice you have in your "die pool" at once.

Every time one of your dice comes up "6," add another six-sided die to your "die pool."

a. Play this game until you reach 11 dice. Starting from 0 rolls and 1 die, Keep a table of the number of times you've rolled, and the number of dice you have.

b. Graph your data as a scatter plot.

c. Compare your graph to your neighbors'

   i. Explain how the graphs are similar

   ii. Explain how the graphs are different

d. Combine your data with two of your neighbors. Create a graph of the average number of dice each person had for each roll.

   i. How does this graph differ from your own graph?

e. Combine all the data in the class: Create a graph of the average number of dice each person had for each roll.

   i. How does this graph differ from your own graph?

   ii. How does this graph differ from the graph of you and your two neighbors?

   iii. The population of the world is about 7 billion. If you combined the data of 7 billion people, what would the graph look like?

Part VII – Reflection

1) What was similar about parts I-VI?

2) What was different about parts I-VI?

3) What is unique about each part?

   a. Part I – Geometric
   b. Part II – Compound Interest
   c. Part III – Per-capita
   d. Part IV – Waiting time
   e. Part V – Doubling Time
   f. Part VI – Stochastic Process

4) What mathematics did you need to know in order to complete each part?

   a. Part I – Geometric
   b. Part II – Compound Interest
   c. Part III – Per-capita
   d. Part IV – Waiting time
   e. Part V – Doubling Time
   f. Part VI – Stochastic Process

5) For each part, identify a conclusion that you can reach using that idea that is much more difficult to reach using the idea of any of the other parts.

   a. Part I – Geometric
   b. Part II – Compound Interest
   c. Part III – Per-capita
   d. Part IV – Waiting time

    e. Part V – Doubling Time
    f. Part VI – Stochastic Process

6) How would you imagine using the ideas of each part in your classroom?
    a. Part I – Geometric
    b. Part II – Compound Interest
    c. Part III – Per-capita
    d. Part IV – Waiting time
    e. Part V – Doubling Time
    f. Part VI – Stochastic Process

7) What obstacles might there be to using the ideas of each part in your classroom?
    a. Part I – Geometric
    b. Part II – Compound Interest
    c. Part III – Per-capita
    d. Part IV – Waiting time
    e. Part V – Doubling Time
    f. Part VI – Stochastic Process